PHL 283
Symbolic Logic (Formal Logic II)
Winter 2016
Depaul University

Instructor: Daniel Rosiak,

Time and Location: Tuesdays, Thursdays 11:00-12:50pm, 2352 N Clifton (the Philosophy building), Room 145

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Overview and Objectives

This course is designed to build on the student's prior knowledge of the basics of symbolic (formal) logic, but the first two weeks will be spent reviewing propositional logic, quantifiers, and the basics of (first-order) predicate logic.[1] We will begin the course with an overview of some of the fascinating topics we will cover in this course. The following few weeks are devoted to reviewing and strengthening your knowledge of propositional and predicate logic, as well as covering a few conceptual and methodological fundamentals. Especially in the first half, emphasis will also be placed on proficiency in proofs, and we will also spend a week looking at some of the main metalogical results in propositional and first-order logic, including soundness, completeness, compactness, and a few others. In the course of introducing these metalogical notions, the student will learn some of the basics of set theory, will be exposed to some closely related idea from other fields (like Boolean algebras in math), as well as to some of the philosophical questions surrounding set theory and a more algebraic approach.

The second half of the course will be devoted to a handful of fascinating special topics and to some applications to interesting philosophical conundrums. This should be very exciting material for the student (especially the student of philosophy). First we will look at a few philosophical motivations for turning to non-classical logics, for admitting more than two truth-values, and we will introduce modal operators. The rest of the course (weeks 7-11) will be devoted to a variety of modal logics and applications. The detailed course calendar below describes each of these class meetings. During this meetings, we will look at time travel and different models of time; different notions of 'necessity' and 'possibility'; deep paradoxes that go to the core of logic and philosophy, such as the sorites paradox, and whether a consistent reasoner can *believe* that they are consistent (without becoming inconsistent). See the course calendar (Special Topics (Weeks 6-11)) for more details.

[1] Because we will review these matters, an ambitious student who feels comfortable learning these matters in a few weeks (it is possible), or who has already learned about them on their own, is invited to take the course with my permission; however, the typical student would have already taken Symbolic Logic I.

Requirements

- **Exercises**: you will be responsible for completing problem sets as homework for *each class meeting*, which together make up **40**% of your final grade in the course
- **Midterm** on 02/18 (**25**% of final grade)
- **Final Project** due on 03/23 (**25**% of final grade); details of this assignment are up on D2L and will be discussed in class
- **Participation/Attendance** (**10**% of final grade)

Course Materials

- There are no required textbooks for this course. All readings will be made available to you on D2L. Some of these readings include selections from:
  - Hausman A., Kahane H., Tidman P. *Logic and Philosophy: A Modern Introduction*. Cengage Learning, 12th edition.
  - Girle, Rod. *Modal Logics and Philosophy*. McGill-Queen's University Press, 2nd edition.
  - Priest, Graham. *Logic: A Very Short Introduction*. Oxford Very Short Introductions, 1st edition.
  - Priest, Graham. *An Introduction to Non-Classical Logic: From If to Is*. Cambridge University Press, 2nd edition.
  - Haack, Susan. *Deviant Logic, Fuzzy Logic: Beyond the Formalism*. University of Chicago Press, reprint.
  - Smullyan, Raymond. "Logicians Who Reason About Themselves." Theoretical Aspects of Reasoning About Knowledge, Proceedings of the 1986 conference (1986): 341-352.
  - My "Lecture Notes."
  - My "Slides on Many-valued Logics."

Course Outline

Note: The readings and exercises are the homework for each designated class meeting—so an Exercise and Reading listed under the Meeting 1 heading are to be ***completed*** *by the beginning of the next class meeting, Meeting 2.*

Review and Basics of Propositional and Predicate Logic (Weeks 1-2)

- Meeting 1: Overview of Course; Review of propositional logic

  Reading: "Propositional Logic Review Sheet" on D2L

  Exercises: answer questions on "Propositional Logic Review Sheet"

- Meeting 2: A little more on propositional logic; Predicate Logic Basics review: quantifiers; domain of discourse; variables, constants, predicates; scope; free vs. bound variables; translations; validity, invalidity, and consistency

  Reading: selections from Chapters 7 and 8 in Hausman (on D2L)

  Exercises: 7-2, 7-5, 7-10 and 8-2 in Hausman

- Meeting 3: The Quantifier Rules and Predicate Logic Proofs

  Reading: Selection from Chapter 9 in Hausman; Copi *Selection* (2 pages) on D2L

  Exercises: questions in Copi Selection

- Meeting 4: More Predicate Logic Proof Practice; Introducing Relational Predicate Logic

  Reading: Chapter 10 in Hausman

  Exercises: 9-5, and 10-1, 10-5 in Hausman

More on Predicate Logic, Tableaux, Some Metalogic, and Limitations of Predicate Logic (Weeks 3-5)

- Meeting 5: Rationale behind the Four Quantifier Rules

  Reading: Chapter 11 (focus on sections 1-3) in Hausman

  Exercises: 11-1 in Hausman

- Meeting 6: Returning to fundamentals and Predicate Logic applied (Set Theory)
  - set theory basics
  - relations and functions
  - Boolean algebras basics
  - the idea of induction
  - brief look at Konig's lemma

  Reading: Selection from Smullyan's *A Beginner's Guide to Mathematical Logic*; Rosiak, "Lecture Notes" (pages 35-43)

  Exercises: questions 3-6 in "Lecture Notes"

- Meeting 7: Revisiting Tree (Tableaux) Method; Propositional Logic Trees

  Reading: Chapters 16 and 17 in Peter Smith's *An Introduction to Formal Logic*, on D2L

  Exercise: Exercise 17 at end of Chapter 17 in Smith

- Meeting 8: Basics of Predicate Logic trees; Some Limitations of First-Order Predicate Logic; Brief Intro to Higher-Order Logics

  Reading: Chapter 12.1-5, and 13.4 in Hausman

  Exercises: question given in class

- Meeting 9: Meta-logic: Soundness and completeness of propositional logic and (brief look at how it works) beyond PL

  - propositional logic trees 'vindicated'
  - tree method is sound
  - tree method is complete
  - quick corollary
  - brief look at how to show soundness and completeness in first-order predicate logic context and more generally

  Reading: Chapter 19 "PL trees vindicated" (in Smith's *An Introduction to Formal Logic*, on D2L)

  Exercise: question given in class

- Meeting 10: Meta-logical results continued (compactness); and the "bigger picture"

  - compactness: the idea
  - Konig's lemma revisited
  - the idea of *decidability*
  - a puzzle

Reading: None...prepare for midterm!

Midterm on 02/18 (covering material from Meetings 1-8)

Special Topics (Weeks 6-11)

Non-Classical Logics Introduced; Some Philosophical Motivations

- Meeting 11: Motivating Non-classical logics; Motivation 1: More than 2 truth values?

  - Philosophical motivations
  - Two 3-valued logics (Kleene and Lukasiewicz) and a 4-valued logic ($\mathbf{B}_4$)

  - A bit about extending to infinite-valued (fuzzy) logic
  - Fuzzy logic applied to *sorites* paradox

  Reading: My "Slides on Many-valued Logics"

  Exercise: answer question about sorites at end of slides

- Meeting 12: Motivation 2: Aristotle's "Future Sea-Battle" problem (introducing the need for modal operators)

  - Introducing the Sea-Battle (Future Contingents) problem
  - Inadequacy of using third truth-value to model this
  - Turn to modal operators

  Reading: Graham Priest's "Necessity and possibility: what will be must be?" (Chapter 6 from *Logic: A Very Short Introduction*, posted on D2L); (optional) Susan Haack's "Future Contingents" (Chapter 4 from *Deviant Logic, Fuzzy Logic: Beyond the Formalism*, on D2L)

  Exercises: questions at end of Priest's Chapter 6

Modal Logics

- Meeting 13: Basics of Modal Logic

  - adding to propositional logic the modal operators $\Diamond$ ("diamond") and $\Box$ ("box")
  - $\Diamond$ as possibility; $\Box$ as necessity...introducing **S5**
  - trees (tableaux)
  - beginning with trees and "world" semantics for **S5**—a diagrammatic way of searching for counter-examples
  - closing a tree, open branches, and locating counter-examples (counter-models) in **S5**

  Reading: Girle's "A simple modal logic" (Chapter 2 in *Modal Logics and Philosophy*, on D2L)

  Exercises: 2.3 (questions 1 and 2), 2.4 in Girle

- Meeting 14: More on normal modal logics

  - extending to other, 'weaker' normal modal logics
  - accessibility
  - **K** as the weakest normal modal logic
  - dropping properties of "accessibility" relation (like transitivity, symmetry, etc.) to get new tree rules (and so new logics)

  - a quick note about 'non-normal' modal logics

  Reading: Selections from Girle's "The normal modal logics" (Chapter 3 in *Modal Logics and Philosophy*, on D2L)

  Exercises: 3.3 questions 1 (all) and 2 (a-c) in Girle

- Meeting 15: Some Philosophical Issues surrounding multiplicity of normal modal logics
  - thinking harder about the various notions of necessity (and (im)possibility)
  - logical necessity
  - metaphysical necessity
  - physical necessity
  - epistemic necessity
  - alethic necessity
  - obligational (moral) necessity

  Reading: Priest's "Which System represents Necessity?" (pages 46-49 in *An Introduction to Non-Classical Logic*, on D2L)

  Exercise: question given in class

Two Extended Applications of Modal Logics

(1) Time Travel; Is time real?

- Meeting 17: Temporal (Tense) Logic and Time Travel
  - Introducing Temporal Logic
  - $\Diamond$ as "at some time in the future," and $\Box$ as "at all times in the future"
  - reinterpreting $\mathbf{K}_4$ as a logic for (past) time
  - a 'dual' logic

  Reading: Selections from Girle's "Temporal Logic" (pages 151-155 in *Modal Logics and Philosophy*, on D2L)

  Exercise: question given in class

- Meeting 18: More on variation on the flow of time
  - Different relations (dropping transitivity, making accessibility irreflexive, etc.)
  - linear time
  - circular time
  - non-circular time

- beginnings and endings

Reading: Selections from Girle's "Temporal Logic" (155-164 in *Modal Logics and Philosophy*, on D2L)

Exercise: finish proof from class

- Meeting 19: Temporal Logics applied to Philosophical Problems
  - Paradoxes of time travel!
  - watch and analyze clip from the sci-fi film *Primer* (2004)
  - a little on whether time is 'real' (McTaggart summarized)

Reading: Selections from Girle's "Temporal Logic" (164-171 in *Modal Logics and Philosophy*, on D2L)

Exercise: watch all of *Primer* (YouTube); and (optional) "Time Travel in Fiction Rundown" (YouTube)

(2) Belief, Provability, and whether a consistent reasoner can (consistently) *believe* that they are consistent

- Meeting 20: Background on Provability, Consistency, (Un)decidability, and Belief Logics
  - introducing concept of provability
  - recalling and revising notions of consistency and (un)decidability
  - $\Box$ re-interpreted as "it is believed that"
  - reinterpreting non-reflexive $\mathbf{S}_4$ in terms of belief
  - introducing Smullyan's doxastic logic and overview of the argument

Reading: Smullyan's "Logicians who reason about themselves" (pages 341-345), here:

`http://www.tark.org/proceedings/tark_mar19_86/p341-smullyan.pdf`

Exercises: try the problems (problem 1, 2, 2A) in Smullyan's essay (he supplies the answers right there; write down your attempts to the problems first, then check your answers against his, and mark your answer)

- Meeting 21: Epistemic counterpart of Godel's theorem
  - The Godel consistency problem applied to reasoners
  - finish covering Smullyan's argument

Reading: Smullyan's "Logicians who reason about themselves" (pages 345-351); and also skim Wikipedia page on Doxastic logic (covering key concepts from Smullyan's essay), here:

`https://en.wikipedia.org/wiki/Doxastic_logic`

Exercises: in at least 3 paragraphs, defining terms carefully, answer the following:

1. describe what the "stability predicament" is; and
2. describe the argument that if a consistent ("stable") reasoner (of type 4) believes that they are stable, then they become inconsistent.

Final Assignment Due 03/23 (covering material from second half of course)