# Fundamentals of Arguments, Important Distinctions and Concepts

Daniel Rosiak

Depaul University

# General Comments

- What distinguishes a properly ethical theory or approach to a situation from "any old" response or reaction?
- An ethical/moral theory provides a systematic framework for forming *judgments backed by reasoning*.

# General Comments

- ▶ Moral/ethical *situations* are ...?
- ▶ ...any situation (actual or hypothetical), that at least implicitly involves some sort of "ought" or "should."
- ▶ So, for instance, the proposition "I like bread" or "The universe is 14 billion years old" would not fall within the ambit of ethics, since neither of these statements imply any sort of "ought."

## General Comments

1. What makes for an *ethical theory* is that it involves a rational defense of a course of action or response.

   If you ask (yourself or others) *why* you "ought to do such-and-such," good reasons, backed by sound logical reasoning and careful consideration of the relevant factors and alternatives, can be supplied.

Captures what Rachels calls the "minimum concept of morality":

> *Morality is, at the very least, the effort to guide one's conduct by reason—that is, to do what there are the best reasons for doing—while giving equal weight to the interests of each individual/entity affected by one's decision.*
>
> *The conscientious moral agent is then someone who is concerned impartially with the interests of everyone/everything affected by what he or she does; who carefully sifts facts and examines their implications; who accepts proposed principles of conduct only after scrutinizing them to make sure they are justified; who is willing to "listen to reason" even when it means revising prior convictions; and who, finally, is willing to act on the results of this deliberation. (Rachels, 13)*

# General Comments

1. So, clearly it is important that we all understand at least the "basics" about what *reasoning* or *argumentation* is, and what makes something count as "good reasons," before attempting to sort out disputes between different ethical approaches.

2. After all, it is pretty uncontroversial that there are bad arguments and there are good arguments. Assessing a particular ethical theory, it is important that we can tell the difference, and that we share a language and set of tools for detecting such a difference.

# General Methodological Remark

Even though ethical theories are at bottom motivated by *practice*—by the need to guide us in actual situations and applications that call upon our ethical faculties—in reasoning *about* ethical theories themselves, i.e., in attempting to adjudicate between differing systematic approaches, we often consider *hypothetical* situations and perform "thought experiments."

This does not make things "less applicable" or "merely theoretical," but is a highly useful way of getting some perspective on the consistency and full *range of applicability* of a systematic ethical approach (or any of its proposals regarding what we should do in a particular situation).

# Important Concepts for Arguments

Reasoning is fundamentally about making *inferences*.

### Definition
To *infer*, or make an *inference*, is to draw conclusions from (a string of) premises.

*You can think of this, for now, as a process that produces an output—the conclusion—given certain inputs—the premises.*

In making inferences, both the premises and conclusions will be *statements* or *propositions*, where this means they are (or are resolvable into) things that are of the 'type'

> *declarative sentences that are either true or false (or, if we don't know, are at least the sort of thing that could be true or false).*

E.g., 'The cat is on the mat'
'$2 + 2 = 4$'
'I am the king of France'
'We live in a multiverse'.

Many sentences we encounter on a daily basis are *not* statements in this sense, for instance: "Close the door, please."

## Definition

▶ A *premise* is a statement that provides reason or support or evidence for a conclusion. An argument may have one or many premises.

  \*If an argument is presented linearly, premises typically come at the beginning, before the conclusion.\*

▶ A *conclusion* is a single statement in an argument that is meant to follow, or be inferred, from the given premises. It usually (but may not) comes at the end of an argument.

  \*\*A conclusion is usually immediately preceded by words such as "therefore" or "thus," but it of course need not be, so you may have to do a little work to parse what is a premise and what is a conclusion.\*\*

### Definition
An *argument* is a collection or sequence of statements, one of which is designated as the conclusion, the rest of which are premises.

*Note that this does not define a *good* argument, just an argument.*

**Note also that the word "argument" is colloquially used to mean a disagreement, usually an unpleasant one, e.g., "She argued with her husband in public." This is *not* the sense of argument we mean here (or in general, in this course).**

# Examples of Arguments

### Example

All humans are mortal. (Premise 1)
Socrates was a human. (Premise 2)
Therefore, Socrates was mortal. (Conclusion)

### Example

There is smoke. (Premise)
So, there is a fire. (Conclusion)

### Example

Grass is green. (Premise 1)
The sky is blue. (Premise 2)
$2 + 2 = 4$. (Premise 3)
Therefore, coal is black. (Conclusion)

# Non-Example

On the other hand, the following is *not* an argument:

## Example

1. Are you hungry?
2. Shut the door!
3. Therefore, help me!

since none of the constituent English sentences are *statements* (they are not *capable* of being true or false).

However, note that *some* sentences, once suitably rephrased, might indeed amount to statements (even if they don't *appear* to be).

- ▶ Notice how in the previous three examples of arguments, there are some obvious differences between the three.
- ▶ There is the obvious difference between the first two (which both appear to be good, or at least plausible) and the third (which appears to be nonsense).
- ▶ But there is also an important difference between the first two, i.e., arguments like the Socrates-Mortality one and the Smoke-Fire one.

### Example

There is smoke. (Premise)
So, there is a fire. (Conclusion)

▶ The existence of smoke does not *guarantee* the existence of fire—it simply makes the existence of fire *likely* or *probable*, since our experience tells us that 'where there is smoke, there is (almost always) a fire.'

▶ So even if the inference is reasonable, based on our experience, it is nonetheless fallible. It is indeed *possible* for there to be smoke without there being a fire, so, however unlikely, we might be wrong to draw the conclusion we did.

▶ The investigation of such inferences is called *inductive logic*.

# Induction

### Definition
*Induction* concerns the process of drawing *probable* (likely, plausible), but ultimately fallible, conclusions from premises or a particular sample—i.e., whenever the truth of the premises *makes likely* the truth of the conclusion.

*Note: it is also accordingly frequently called *probabilistic reasoning*.*

It is exemplified by arguments like the following:

### Example
These beans are from this bag. (Premise 1)
These beans are white. (Premise 2)
All the beans in this bag are white. (Conclusion)

# Deduction

### Definition
*Deductive logic* concerns inferences such as the Socrates example, where, as long as the premises are in fact true, then the conclusion is *certainly/necessarily* true (cannot be otherwise). In other words, the truth of the premises *guarantees* the truth of the conclusion.

As such, deduction is a rigorous and exact science.

Deductions are of things like the following:

### Example
All the beans in this bag are white. (Premise 1)
These beans are from this bag. (Premise 2)
These beans are white. (Conclusion)

# Abduction

A third type of reasoning/argument is given by *abduction*. It is closer to inductive reasoning than it is to deductive reasoning.

## Definition
*Abduction* is a form of logical inference that proceeds from observations to a conclusion that is a hypothesis that would account for the data or the truth of the premises.

*Note: as with induction, the conclusion is thus *at most* plausible.*

## Example
All the beans in this bag are white. (Premise 1)
These beans are white. (Premise 2)
These beans are from this bag. (Conclusion/Hypothesis)

# Stepping Back

▶ In reading, discussing, and forming arguments in this course, we will encounter many examples of all three argument types.

▶ However, the study of arguments of the inductive and abductive type is actually fairly complicated, and many of the pivotal concepts in logic that are useful to thinking about reasoning in general come from the study of deductive logic.

▶ So let's look a little closer at some of these concepts and distinctions arising from the study of deductive logic.

# Form vs. Content

We need a couple more distinctions and definitions...

### Definition
The *form* of an argument is the order/arrangement of such abstract terms, what makes it valid or invalid. The *content* is supplied by whatever statements we substitute in for such variables or terms.

### Example
All humans are mortal. (Premise 1)
Socrates was a human. (Premise 2)
Therefore, Socrates was mortal. (Conclusion)

This has the form

### Example
All $H$ are $M$.
$S$ is an $H$.
Therefore, $S$ is $M$.

# Validity and Invalidity

### Definition

▶ An argument is said to be *valid* provided formally, i.e., in terms of its form, the conclusion follows from its premises.

▶ This means that it is impossible for the conclusion to be false provided the premises are all true, i.e., there can be *no counterexample*, no instance or substitution with true premises and a false conclusion.

▶ An argument is *invalid* otherwise. In other words, when the form of an argument is invalid, the truth of the premises cannot guarantee the truth of the conclusion.

# Example of an Invalid Argument

## Example

All popes reside at the Vatican. (Premise 1) (true)
Francis resides at the Vatican. (Premise 2) (true)
Therefore, Francis is a pope. (Conclusion) (true!)

However, resolving this argument into its form, then producing one of the many instances/substitutions that make the premises true but the conclusion false, shows that it must be invalid:

## Example

All $P$ are $V$. (Premise 1)
$F$ is a $V$. (Premise 2)
Therefore, $F$ is a $P$. (Conclusion)

Make the following substitution:

## Example

All basketballs ($P$) are round ($V$). (Premise 1) (true)
The Earth ($F$) is a round thing ($V$). (Premise 2) (true)
Therefore, the Earth ($F$) is a basketball ($P$). (Conclusion) (false!)

## In Brief...

An argument that is valid is one that has "good form," while an invalid argument one that does not.

# Common Valid Argument Forms

Some (certainly not all!) common valid argument forms include:

- ▶ Modus ponens:
  If P, then Q.
  P.
  Therefore, Q.

- ▶ Modus tollens:
  If P, then Q.
  ¬ Q ('not' Q).
  Therefore, ¬ P.

- ▶ Disjunctive syllogism:
  Either P or Q.
  ¬ P.
  Therefore, Q.

- ▶ Hypothetical Syllogism:
  If P, then Q.
  If Q, then R.
  Therefore, if P, then R.

# Soundness vs Validity

The notion of validity is to be distinguished from the following:

### Definition
A *sound* argument is one that is *both* valid and whose premises are all factually correct or true.

In brief: a sound argument has both "good form" and "good content."

To appreciate this distinction, note that the following is in fact a perfectly *valid* argument, even though it is obviously not sound:

### Example

The President is a B-52 Bomber. (Premise 1)
I am the President. (Premise 2)
Therefore, I am a B-52 Bomber. (Conclusion)

This has "good form" and so is valid. Valid arguments can have false premises (and false or true conclusions), as in the above, where none of the premises (or conclusion) happen to be true.

On the other hand, some valid arguments *do* have true premises—and by virtue of validity, whenever this is the case, it must have a true conclusion as well: such arguments are sound.

# Important Take-Away

Valid arguments all of whose premises are true are said to be *sound*, which is thus *stronger than validity*.

While it will be important to be able to tell whether or not an argument is valid, we of course ultimately want to produce (and accept) only arguments that are *sound*.

# Some Last Definitions!

### Definition

▶ Logical *operators*, like ¬ ('not'), ∧ ('and'), → ('implies', or 'if...then'), etc., are used to build up compound sentences from simpler components. Depending on the chosen logic, there are "truth tables" supplied for the operators, that tell how to determine the truth value of the compound given certain truth values assigned to the components.

   *Don't worry about this; I just mention it for completeness.*

▶ A *contradiction* is a logical incompatibility between two or more statements, i.e., the statements, taken together, yield two conclusions which are opposites (*A* and not-*A*).

▶ At the opposite extreme of a contradiction stands a *tautology*: a formula that is *always true*, i.e., true for every assignment of truth values. E.g., any statement of the form '*A* or not-*A*' is a tautology.

▶ A *conditional* is a statement of the form 'if...then'.

# Consistency and Inconsistency

### Definition
A collection of statements is *logically consistent* provided they can all be true together (meaning: don't contradict each other).

A collection is *logically inconsistent* provided they cannot all be true at the same time (meaning: at least two of the statements contradict one another, or lead to contradictory conclusions when asserted together).

# See the Handout for A Few More

A couple other concepts (really only two or three) that may come in handy in reading philosophy and critically examining arguments—like *necessity* and *sufficiency* in conditionals—are defined in the "Basics Concepts: Ethics" (Rosiak) document on D2L, which you can feel free to consult as needed.

# Stepping Back

Okay, fine...now we have lots of definitions.

But what can we *do* with these, and what does it all have to do with *ethics*?!

Later, we will see some of the ways of using these concepts and tools to help us *construct* good arguments (which we said was a pivotal part of what ethical theory was about).

But first we will look at how these notions can be used to *diagnose* or detect bad arguments (which is just as important, since we need to be able to tell the good reasons and arguments from the bad).

# Bad Arguments: Fallacies

## Definition

▶ A *fallacy* is the use of invalid or otherwise "broken" reasoning in the construction of an argument.

▶ While some fallacies are simply invalid arguments (once parsed), fallacies can be a little more subtle. Sometimes one uses the term fallacy to capture "informal fallacies," which involve errors of reasoning other than those that rely on an argument with an invalid *logical form*, usually involving misuse or complications regarding the content of the argument.

▶ Being aware of common fallacies helps us be on the look out for *bad* or unpersuasive arguments, even when the form appears to be logically acceptable.

# 18 Common Fallacies to Watch Out For

1. **Ad hominem** ("attack on the person"): this substitutes a personal insult or attempt at undermining a person (or their intelligence, morals, education, qualifications, character, etc.) for a genuine rebuttal of the actual argument put forward by the person.

   ### Example
   Eddie's arguments on education cannot be correct, since he didn't even finish high school.

   Notice how this does not even address or evaluate the actual arguments proposed by Eddy.

2. **Affirming the Consequent**

## Example

If Jane is a Marxist, then she does not support capitalism (If A, then B).

Jane does not support capitalism. (B)

Therefore, Jane is a Marxist. (A)

This is a fallacious argument! To see this clearly, make another "substitution" into the same *argument form*:

## Example

If it just rained, then the grass is wet. (If A, then B).

The grass is wet. (B)

Therefore, it just rained. (A)

No! The conditional (first line) is certainly true. And the second premise ('The grass is wet') might very well be true also, without it being true on account of it having rained. (Perhaps someone watered the grass, or the neighbors set up a slip-and-slide.)

# A Nuance

However, if you have been paying very close attention, you may have noticed that the previous fallacy is closely related to the form of an *abduction* (which we spoke of as if it were a legitimate form of reasoning, and it is!).

So there might be instances/situations where such a type of argumentation (to a probabilistic conclusion) is appropriate. However, if the argument is meant to be *deductive*, leading to a conclusion that is plainly necessarily true, then arguments of this form are invalid, making it fallacious.

This shows that it pays to be conscious of what type of argument you are dealing with (deductive, inductive, abductive)!

3. **All-or-Nothing**: whenever one assumes that one must commit to all or nothing of a particular collection of beliefs.

Example

"You say that there is no such thing as Hell, but then how do you account for the evidence that Jesus was a real person?"
(This assumes, wrongly of course, that if you reject one belief of Christianity (the existence of Hell), then you must thereby be rejecting every other belief held by Christianity.)

4. **Ambiguity/Equivocation/Vagueness**: when a conclusion is drawn from premises that are vague/ambiguous/equivocal.

<span style="color:green">Example</span>

Notes are written by musicians.
I am writing my mother a note.
Therefore, I am a musician.


'Note' is used equivocally here (has multiple meanings).

5. **Anecdotal Evidence**: using anecdotal (highly particular) evidence as support for a more general or universal conclusion.

   Example
   1. Jonny married his high-school sweetheart.
   2. Jonny's high-school sweetheart is now on trial for murder.
   3. Therefore, you should never marry your high-school sweetheart.

6. **Appeal to Authority/Popularity/Popular Belief/Tradition**: anything of the following form:

Example
Most people (or $X$ tradition/authority) believe $A$ to be true.
Therefore, $A$ must be true.

# Meta!

7. **The Fallacy Fallacy** (or "argument from fallacy"): assumption
   that if a *given argument* for some conclusion proves to be
   fallacious, then the *conclusion* must be false. Of course, this need
   not be the case.

   ### Example

   Sarah said that most people believe $A$ to be true, and concluded
   that $A$ must be true.
   What Sarah did was a fallacious form of reasoning.
   Therefore, $A$ cannot be true.

   No! Maybe $A$ *is* true! Just because Sarah gave a bad argument for
   $A$ does not automatically make it false. Might seem obvious, but
   it's important!
   Perhaps there *is* a good argument out there—Sarah just didn't
   know about it, or we just haven't found it yet.

8. **Begging the Question/Circular Reasoning**: tacitly *assuming* (adopting as a premise), instead of *supporting*, the very thing you aim to conclude.

   ## Example

   Everyone wants the new IPhone, because it is the hottest item on the market!

   Sometimes, such fallacies are harder to detect, as in the following sentence from a published document:

   > *To allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the State, for it is highly conducive to the interests of the community that each individual should enjoy a liberty perfectly unlimited of expressing his sentiments.*

9. **False Analogy**: assuming that because two things are alike in one or more respects, they are alike in some other (or every) respect.

## Example

A medical student says "No one objects to a doctor looking up a difficult case in medical books. Why, then, shouldn't medical students taking a difficult exam be allowed to consult their medical textbooks?"

10. **Hasty Generalization** (or "argument from small numbers"):
    drawing a conclusion based on a small sample size, or concluding a
    general claim from limited evidence.

    ## Example

    1 is a square number.
    3 is a prime number.
    5 is a prime number.
    7 is a prime number.
    9 is a square number.
    11 is a prime number.
    13 is a prime number.
    Therefore, all odd numbers are either prime or square.

    No! 15 is a counterexample (neither prime nor square)!

11. **Is-Ought**: assuming that because things happen to be (or have been) a certain way, they *ought to be* that way.

Example

It is a fact that human beings have always been at war with one another, throughout recorded history, i.e., conflict is a basic fact of human history. Thus, war must be good for human beings.

12. **Non-sequitur** (latin for "it does not follow"): an irrelevant premise or scenario is offered in support of a conclusion.

More generally, it is a conclusion or reply (to an argument) that does not follow, logically, from the string of preceding statements. Sometimes this just amounts to adopting an invalid logical argument form, such as "affirming the consequent" or (its dual)

### Example

Good people don't lie.
You told a lie.
Therefore, you are not a good person.

The conclusion *does not* follow!

### Example

Adolf Hitler was a vegetarian. Hitler was evil. Therefore, being a vegetarian is evil.

Funny and self-referential joke *about* non-sequiturs from Bill Griffith (which "definition" itself acts out a non-sequitur):

> *Non-sequitur (definition): when a train of thought proceeds from A to B and back again to Q.*

13. **Secundum Quid**: a deductively valid argument that is unsound (but passed off as sound), by specifically confusing what is true *in a certain respect* (or only when subject to certain decisive contextual markers) for what is true *absolutely* (or *unconditionally* or in full generality).

In short: it ignores important qualifications (hence its other common names: "ignoring qualifications" or "destroying the rule").

### Example

Did you see that ambulance run that red light just now? Clearly, you are allowed to drive however you want around here.

### Example

Cutting people with knives is a crime.
Surgeons cut people with knives.
Therefore, surgeons are criminals.

# More on the Above

### Example

Cutting people with knives is a crime.
Surgeons cut people with knives.
Therefore, surgeons are criminals.

Notice that this is formally valid! However, the second premise—"Surgeons cut people with knives"—is really a statement that is only true subject to certain very important qualifications.

The argument only *seems* to work because it subsequently proceeds to erase this decisive qualification, and treat the surgeon's cutting as the same as unqualified "cutting people with knives."

These examples explain why Aristotle, who first described this fallacy, called it the *fallacy of unqualified generalization*.

14. **Slippery Slope**: assuming that by allowing $X$, we would be inviting some (plausible) relatively small or initial consequence (that allegedly follows from or goes along with $X$), and this would be to allow an *entire chain of related events/consequences*, ultimately culminating in a non-small (non-negligible) effect.

    The fallaciousness of this argument is related to another fallacy—called the *continuum fallacy*—by ignoring the possibility of a middle ground or continuous/gradual transition, instead assuming a discrete transition.

    ### Example

    "This week it's Robert E. Lee. I notice that Stonewall Jackson's coming down. I wonder; is it George Washington next week, and is it Thomas Jefferson the week after? You know, you really do have to ask yourself, where does it stop?" [Donald Trump discussing the removal of Confederate statues]

The above example is certainly a case of a *fallacy*.

However, most logic and critical thinking textbooks acknowledge that "slippery slope style" arguments can be tricky, and can even sometimes be decent (inductive/abductive) arguments:

> *slippery slope arguments can be good ones if the slope is real—that is, if there is good evidence that the consequences of the initial action are highly likely to occur. The strength of the argument depends on two factors. The first is the strength of each link in the causal chain; the argument cannot be stronger than its weakest link. The second is the number of links; the more links there are, the more likely it is that other factors could alter the consequences. (Kelly, The Art of Reasoning)*

Moral: one should be a little careful in "diagnosing" arguments as fallacious when they are of the "slippery slope" type.

15. **Straw man**: giving the *impression* of refuting someone's argument, but actually first substituting a superficially similar, but actually different (and less plausible or sound) argument, and then refuting that.

   One basically first recasts or misrepresents the argument in a distorted way, and then takes down this modified version.

   ### Example

   Zebedee: What is your view on the Christian God?
   Tom: I don't believe in any gods, including the Christian one.
   Zebedee: So you think that we are here by accident, and all nature is pure chance, and the universe just created itself?
   Tom: You got all that from me stating that I just don't believe in any gods?

# Unpacking the Previous Exchange

In the above exchange, Tom simply claimed that he did not believe in any Gods, the Christian one in particular. From that, one can deduce he is not a theist, and probably fairly assume certain other believes (like that he does not believe in "life after death").

However, it is not at all apparent how from not being a theist we could deduce that Tom believes that we are all here "by accident" or any of the other extreme things Zebedee immediately attributes to him.

**This fallacy is sometimes *opposed* to a (good) type of argument called *steelmanning*: the idea of this is to first find the *best form or showing* of an opponents argument and then evaluate this "improved" version. Socrates was famous for this.**

16. **Tu Quoque** (latin for "you too"): any attempt to discredit an argument by asserting that the argument's proponent has failed to act consistently in accordance with its conclusions.

*It is more or less a special case of the ad hominem fallacy discussed above.*

### Example

Mother: You should stop smoking, since it is harmful to your health.
Daughter: Why should I listen to you? You started smoking when you were 16!

Even if the mother continues to smoke, and so may indeed be failing to act in accordance with her own advice, this does not invalidate the (implicit) argument that smoking harms one's health.

17. **Victory by Definition** (Appeal to the Dictionary): using a dictionary's definition to support an argument or position about the nature of the concept named by that word.

    This is a fallacy since *dictionaries do not reason*—they simply reflect an abbreviated version of the current accepted popular usage of a term, as dictated by eventual acceptance into speech patterns. Dictionaries do not make any claims as to what the underlying concept (which may be named by that word) means or ought to mean.

    ### Example

    "The dictionary defines the *mind* as 'the element of a person that enables them to be aware of the world and their experiences, to think, and to feel; the faculty of consciousness and thought.'" (From a student's philosophy paper)

18. **Wishful Thinking**: any argument one (or more) of whose premises expresses a *desire* (or implicit bias) for the conclusion to be true or false, and aims to pass from this wish to it *actually being* true or false.

    *It is not the same as "positive thinking" (which can actually be useful and rational in certain circumstances). Rather, it is what happens when one lets one's beliefs or arguments be based more on what is pleasing or comfortable to imagine than on what the evidence or reason suggests is in fact the case.

    ### Example

    If there were no life after death, then this life would seem meaningless.
    (Implied further premise) I don't wish to believe that this life is meaningless.
    Therefore, I choose to believe that there is life after death.

# General Tips/Steps for Examining an Argument

No recipe, but general things to do:

1. Identify the premises, the conclusion, and the logical form of the argument
2. Be sure you know if the argument is supposed to be deductive or non-deductive (of one of the other two types)
3. Determine whether the argument is valid
4. Determine whether there are any "hidden assumptions" and tease these out. (And try to "steelman," not "strawman"!)
5. If it is valid, assess whether it is also sound, by assessing the truth or plausibility of the premises/assumptions.
   In performing this step, you may need to consider further critical questions, such as
   - ▶ If the premise appeals to expertise: how credible/unbiased is the expert source appealed to by the premise?
   - ▶ How strong is the evidence appealed to in the assertion? (Evidence does not speak for itself.)
6. Decide whether the argument is good/bad "as is" or whether it needs to be qualified in some way, in order to become good.

Your best bet in evaluating an argument is to first follow some version of the above steps and, in doing so, evaluate whether or not the argument contains some fallacy. However, beyond this, there are some additional useful methods for evaluating or rebutting or improving an argument, and even for constructing your own (good) arguments, some of which are given below.

# A Few Common (Counter)Argument Methods

1. **Counterexamples**: showing invalidity by supplying *counterexamples*.

   Recall from the above discussion of form and validity that no argument with all true premises but a false conclusion can be valid. Thus, if an argument has all true premises but a false conclusion, it must be invalid.

   So...to show that an argument is invalid, it would suffice to produce "substitutions" into the relevant argument form that yield all true premises but a false conclusion. (Note that this simultaneously proves that both the particular argument (substitution) and the entire logical form deployed by the argument are invalid.)

# Anatomy of a Counter-Argument by Counterexample

### Example

If God exists, then life has meaning.
But there is no God.
Therefore, life is meaningless.

- ▶ Step 1: **Translate into a more exact logical form**:
  1. If God exists, then life has meaning.
  2. It is not the case that God exists.
  Thus, 3. It is not the case that life has meaning.
- ▶ Step 2: **Isolate terms**: Set $G$ = 'God exists'; $L$ = 'life has meaning'. Then we have:
  1. If $G$, then $L$.
  2. Not $G$.
  Thus, 3. Not $L$.
- ▶ Step 3: **Supply Substitution**: now find a substitution making the two premises true and the conclusion false. Try, for instance, substituting $G$ = 'I am dead' and $L$ = 'I am a U.S. Citizen'...

Then, we have
1. If I am dead, then I am a U.S. Citizen. (This conditional is true!)
2. It is not the case that I am dead. (True)
Thus, 3. It is not the case that I am a U.S. citizen. (False!)

So we have supplied a substitution instance into the same argument form (as the original argument) where both premises are true and the conclusion is false, showing that the argument must be invalid.

2. **Locate hidden or implicit premises**:

   ## Example

   "Murder is always wrong. Even though certain states sanction capital punishment, clearly capital punishment is wrong."

   Hidden premise: 'capital punishment is murder'.

   It doesn't matter if you happen to agree with this—it is not obvious, so it is not something you can take for granted.

   Note: "Locating" the hidden premise does *not* amount to "disproving" the argument. It simply means rewriting the argument to display those premises:

   ## Example

   Murder is always wrong.
   Capital punishment is murder.
   Thus, even though certain states sanction capital punishment, capital punishment is wrong.

   Now the argument can be evaluated.

# The Moral

The point is that *you* should try to be as explicit as possible, and to always spell out all the assumptions, in your own arguments and in your evaluations of those of the authors you read.

In slogan form:

> *In order for the significance and soundness of an argument to be properly assessed, hidden premises, no matter how seemingly "obvious" to you, ought to be spelled out explicitly. Then, and only then, should you assess!*

3. **Reductio ad absurdum** (latin for "reduction to absurdity/the impossible"): a form of (counter)argument that attempts either to
    - ▶ (a) prove a statement by first assuming it were *not* true, and then showing that such an assumption would lead to an impossible or "absurd" conclusion (or to a contradiction); or
    - ▶ (b) disprove a statement by showing that assuming its truth will lead to an impossible or absurd conclusion.

\*\*This technique is massively useful, either as a tool of discovery of true statements and as a means of rebutting claims.\*\*

# First (a)

The former method, (a), is also sometimes called, especially in math and logic contexts, "proof by contradiction," or "indirect proof," since it argues that the denial of the premise results in a logical contradiction (an "impossibility").

### Example

Claim: There is no smallest positive rational number.
*Proof by reductio*: Assume there *is* a smallest positive rational number, call it $s$. (*Note: this is the same as assuming that it is *not the case* that there is no smallest positive rational number.*) But we can divide any rational number by an integer, say by 2, and the result will be another rational number. So consider $r = s/2$. Then $r$ is rational. But $r$ is smaller than $s$. So $s$ is not the smallest rational number.

**Note: we have produced an absurdity/contradiction—*s both is and is not the smallest positive rational number*.**

# Then (b)

For an illustration of (b)—*reductio* that *disproves a statement* by first assuming its truth then showing that this assumption will lead to an impossible or absurd conclusion—consider the following:

### Example

Sam: "The earth is flat."
You (*counterargument by reductio*): "Fine, let's assume the earth *is* flat. Then,

...due to properties and facts concerning the nature of human perception and planes, which we will assume have been granted...

when you watch a large sailboat sailing away from the shore, it should appear to you to get smaller and smaller with the entire sailboat—both the boat and the sail—decreasing in size. But that is "absurd" since this is exactly what we *do not* see—we see the boat fall below the horizon while still seeing the sail.

Thus, your claim that the earth is flat cannot be correct."

4. **Contraposition**: it is a fact of logic that any conditional statement of the form $P \to Q$ ('P implies Q') is "logically equivalent" to its so-called *contrapositive* $\neg Q \to \neg P$ ('not-Q implies not-P'), formed by negating both terms and reversing the direction of inference.

Even though these are equivalent, sometimes it is a lot easier to give a proof of (or evaluate the truth of) one instead of the other.

### Example

Prove the following: '(For $x$ an integer) If $x^2$ is even, then $x$ is even.'
A direct proof is possible (but longer). So let's prove the contrapositive instead: 'If $x$ is *not* even, then $x^2$ is *not* even.'

*Proof*: Suppose $x$ is not even. Then $x$ must be odd. But the product of two odd numbers is odd. Thus since $x^2 = x * x$, the product of two odd numbers, it must be odd. Thus $x^2$ is not even.

*We have shown that 'If $x$ is *not* even, then $x^2$ is *not* even.' We can immediately infer, *by contraposition*, the original statement.*

5. **Removing vagueness/ambiguity**: suppose the argument contains *vague (or equivocal) terms*; resolve this by removing the vagueness, removing the imprecision or ambiguity by making things more explicit, re-defining terms or adding premises as needed.

   Then re-evaluate using the above general methods as appropriate.